QIT1

$A=(1,0,0)$
$B=(0,0,1)$
$C=(1,1,1)$
$D=(0,1,0)$

① $\vec{AC}\cdot\vec{AD}$

$\vec{AC}=\langle 0,1,1\rangle$
$\vec{AD}=\langle -1,1,0\rangle$

$\langle 0,1,1\rangle\cdot\langle -1,1,0\rangle = 0+1+0=\boxed{1}$

② $\vec{AC}\cdot\vec{AD}=|\vec{AC}||\vec{AD}|\cos\theta$

$\cos\theta=\dfrac{\vec{AC}\cdot\vec{AD}}{|\vec{AC}||\vec{AD}|}$

$|\vec{AC}|=\sqrt{0^2+1^2+1^2}=\sqrt{2}$
$|\vec{AD}|=\sqrt{2}$

$\cos\theta=\dfrac{1}{(\sqrt{2})(\sqrt{2})}=\dfrac{1}{2}$

$\cos^{-1}\left(\frac{1}{2}\right)=\boxed{60^\circ}$

3. Find the projection Vector of $\vec{AC}$ on $\vec{AD}$

Projection Vector Formula:

Projection of B on A $\dfrac{\langle A\cdot B\rangle}{\|A\|^2}\cdot A$

$\vec{AC}=\langle 0,1,1\rangle$
$\vec{AD}=\langle -1,1,0\rangle$

$\langle A\cdot B\rangle = 1$

$\|A\|^2=\left(\sqrt{0^2+1^2+1^2}\right)^2=2$

$\dfrac{1}{2}\cdot\langle -1,1,0\rangle$

$=\langle -1/2, 1/2, 0\rangle$

④ Vector $\perp$ to $\vec{AC}$ & $\vec{AD}$ is cross product

$\vec{AC}\times\vec{AD}$

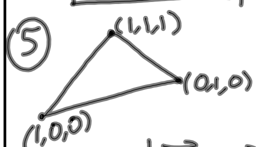
$\langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle$

$=\langle 0,1,1\rangle\times\langle -1,1,0\rangle$

$=\langle 0-1, -1-0, 0+1\rangle$

$=\boxed{\langle -1,-1,1\rangle}$

⑤



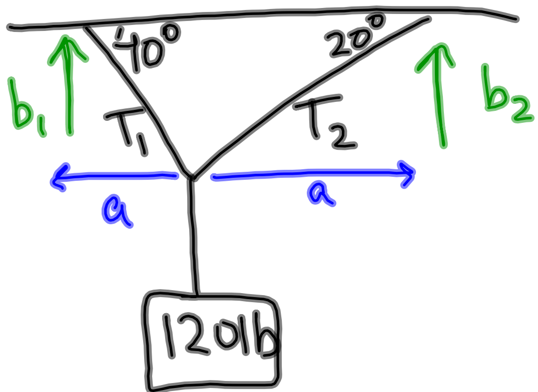
$A_\triangle=\dfrac{|\vec{CA}\times\vec{CD}|}{2}$

$\vec{CA}=\langle 0,-1,-1\rangle$ $\vec{CD}=\langle -1,0,-1\rangle$

$\vec{CA}\times\vec{CD}=\langle 1,1,-1\rangle$

$\dfrac{\sqrt{1^2+1^2+(-1)^2}}{2}=\boxed{\dfrac{\sqrt{3}}{2}=A_\triangle}$

b



$$\tan 20° = \frac{b_2}{a}$$

$$\tan 40° = \frac{b_1}{a}$$

<u>horizontal</u>

$$a = a$$

<u>vertical</u>

$$b_1 + b_2 = wt$$

$$a\tan 40° + a\tan 20° = 120\text{lb}$$

$$a = 99.744 \approx 100$$

$$b_1 = 100\tan 40° = 84$$

$$b_2 = 100\tan 20° = 36$$

$$T_1 : \langle 100, 84 \rangle \quad |T_1| = \sqrt{100^2 + 84^2} = 131$$

$$T_2 : \langle 100, 36 \rangle \quad |T_2| = \sqrt{100^2 + 36^2} = 106$$

Q1T2

① $A(1,2,3)$ $\overrightarrow{AB}=\langle -1,0,-2\rangle$

$B(0,2,1)$ $\overrightarrow{AC}=\langle 1,-2,-2\rangle$

$C(2,0,1)$

$\overrightarrow{AB}\times\overrightarrow{AC}=\langle 0-(-2)(-2),-2-2,2\rangle$

$=\langle -4,-4,2\rangle$

$\langle -4,-4,2\rangle\cdot\langle x,y-2,z-1\rangle=0$

$\boxed{-4x-4y+8+2z-2=0}$

② $2x+y-4z=9 \rightarrow N_1=\langle 2,1,-4\rangle$

$x-y+z=6 \rightarrow N_2=\langle 1,-1,1\rangle$

Plug in valurs ($x=0$ + $x=1$)
to find two points

$A=(0,-11,-5)$

$B=(1,-9,-4)$ ← Not scalar multiples → intersect

$\overrightarrow{AB}=\langle 1,2,1\rangle$

$r(t)=\langle 0,-11,-5\rangle+t\langle 1,2,1\rangle$

③ $r(t) = \langle t, 2\cos t, t\sin t \rangle \quad 0 \le t \le 4\pi$

$y = 2x\cos x \quad z = x\sin x$

$y^2 + z^2 = 4x^2\cos^2 x + x^2\sin^2 x$

traces are ellipses

cones on x-axis